Enrollment No……………

Master of Science (Mathematics)
Third Semester Main Examination, December-2021
Theory of Linear Operators-I [MSM303T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no. 5 has two parts. Part A is 10 marks and Part B is 7 marks.

Q.1 (a) If X be a Banach Algebra and let x∈x the r(x) = $\lim_{n\to\infty} ||x^n||^{1/n}$
(b) Write three properties of Banach Algebra.

OR

(a) Let X be complex Banach division algebra then prove that X is isomorphic to C.
(b) If X is Banach Algebra and x∈x then σ(x)≠ϕ.

Q.2 (a) If T : L_2 [a,b] →L_2 [a,b] be an operator with separable kernel then T is compact
(b) State compact integral operator.

OR

(a) Prove that bounded linear operator O finite rank is compact.
(b) Define compact operator with example.

Q.3 (a) Prove that every linear space X has Hamel Base.
(b) Define normal linear space with example.

OR

(a) Prove that $|< x, y >| \le ||x||.||y||$ When x,y belong to Hilbert space H.
(b) Write any three Spectral properties of Compact linear operator.

Q.4 (a) Let X be a normal space and T:x→x be compact linear operator, if dimX=∞ then show that o∈ σ (T)
(b) What do you mean by Continuous and residual spectra.

OR

(a) Prove that normal linear space is a linear space with respect natural metric defined by d(x,y) = $||x - y||$
(b) Defined closed operators with example.

Q.5 (a) Write spectral properties of bounded linear operator.
(b) What do you mean by Spectrum. Explain with one example.

OR

(a) State and prove Spectral mapping theorem for Polynomial.
(b) Write any three properties of resolvent.

Enrollment No.................

Master of Science (Mathematics)
Third Semester Main Examination, December 2021
Advanced Numerical Analysis-I [MSM304T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Prove that N^{th} Legendre polynomial Pn is given by –

$$Pn(x) = \frac{1}{2n\ N!}\ \frac{d^n}{dx^4}\ [(x^2-1)^n]$$

(b) Prove shifted herby shev first kind

OR

(a) Explain Gram-Schmidt orthogonalization.
(b) Write the formula for che by shev polynomial

Q.2 (a) Find the first derivative at x = 4.5 for the following data.

X	2	4	6	8	17
Y	3	8	8	11	13

(b) Show that E ≡1+Δ or Δ≡ E-1.

OR

(a) Find the first derivative of f(x) at x = 1.5

x	1.5	2.0	2.5	3.0	3.5	3.5
y	3.375	7	13.625	24	38.875	59

(b) Explain Non uniform points.

Q.3 (a) Using the method of liner interpolation find y at x = 0.5 and x = 0.75 given the following table

X	0	1	2	5
Y	2	3	12	147

(b) Explain Quadratic Spline interpolation.

OR

(a) Fit the linear spline to the following data.

X	1	3	6	8
Y	4	5.5	7	9.5

(b) Explain Hermite interpolation.

Q.4 (a) What do you mean by choice of methods.
(b) Evaluate $\Delta^n\ (e^{ax+b})$

OR

(a) Prove that uniform approximation by polynomial when sets one not compact.
(b) Explain rational approximation.

Q.5 (a) Fit a straight line the following data

x	2	4	6	8	10
y	3	7	9	11	17

(b) Explain discreet and continuous data.

OR

(a) Find the value of y at x=0. Given some set of values (-2,5) (1,7) (3,11) (7,34)? Lang ranges interpolation formula.
(b) Write three application of Newton's Bivariate interpolation polynomial.

Enrollment No.................

Master of Science (Mathematics)
Third Semester Main Examination, December-2021
Integration Theory - I [MSM309T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions.

Q.1 (a) What are signed measure ? Explain with example.
(b) State Hahn decomposition theorem.

OR

(a) What do you mean by mutually singular measure Explain.
(b) State Jordan decomposition theorem.

Q.2 (a) If F_1 and F_2 are measure function then the function F^* and F_* are measurable.
(b) Define measure with example.

OR

(a) Prove that every finite measure is semi-finite.
(b) Define sigma finite set with example.

Q.3 (a) Show that constant function with a measurable domain in measurable.
(b) Define step function with example.

OR

(a) Show that the function Ψ defined on R by

$$\Psi(x) = \begin{cases} x+5 & if\ x < -1 \\ 2 & if\ -1 \leq x < 0 \\ x^2 & if\ x > ,0 \end{cases}$$

(b) What do you mean by measurable function.

Q.4 (a) State and prove Lebesque decomposition theroem .
(b) Define Characteristic function.

OR

(a) State and prove Radon Nykodym theorem.
(b) Define mean fundamental.

Q.5 (a) A mean fundamental sequence {fn} of integrable functions is fundamental is measure.

(b) Show that Lébesque measure is Regular.

OR

(a) If F is measurable , g is integrable and $|f| \leq |g|$ Then F is integrable.
(b) Prove that there exists a non-measurable set.

Enrollment No................

Master of Science (Mathematics)
Third Semester Main Examination, December-2021
Functional Analysis-I [MSM301T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Define Linear functional with two example

(b) Prove that is a linear operator is continuous then it is bounded.

OR

(a) Define functional with example.

(b) Define bounded linear operator with example.

Q.2 (a) Explain Banach space.

(b) What is Linear operator? What is difference between a linear operator and linear transformation.

OR

(a) Explain dual space with example.

(b) Prove that every finite dimension normed linear space is a Banach space.

Q.3 (a) Explain complex Vector space.

(b) What do you mean by finite dimensional normed space.

OR

(a) Prove that R^n is an NLS (normed linear space) with the following norms

$\|x\|_1 = \sum_{i=1}^{n} | x_i |$

(b) Explain Quotient space.

Q.4 (a) Defined normed linear space what do you mean by complex normed linear space.

(b) Explain Linear operator.

OR

(a) Let n be a Normed linear space and let ,X,Y ∈ N then

$$\|x\| - \|y\| \leq \|x - y\|$$

(b) Write a short note on Compactness .

Q.5 (a) Show that the real linear space R and the complex linear space C are Banach spaces under the norm.

$\|x\| = |x| \quad ,X \in (R)$

(b) State and prove Riesz lemma

OR

(a) Any Two application of bounded linear functional on c [a,b]

(b) Explain normed space.

Enrollment No..................

Master of Science (Mathematics)
Third Semester Main Examination, December-2021
Advanced Special Function-I [MSM302T]

Time: 3:00 Hrs **Max Marks 85**

Note : Attempt all questions. Each Question has two part has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) State and prove Whipple's Theorem.

(b) State and prove Ramanujan's Theorem.

OR

(a) Define contiguous function.

(b) State and prove Dixon's Theorem.

Q.2 (a) Prove that $\sqrt{1} = 1 \; and \; \sqrt{n+1} = n\sqrt{n} \; and \; \sqrt{y_2} = \sqrt{\pi}$

(b) State and prove Legendre's duplication formula .

OR

(a) Prove that β(m,n) = $\frac{\sqrt{m}\,\sqrt{n}}{\sqrt{m+n}}$

(b) State and prove Gauss multiplication theorem.

Q.3 (a) Find the inverse of x = $\begin{bmatrix} 2 & 4 \\ 5 & -1 \end{bmatrix}$ Hence solve the simultaneous equations

2x+4y = 1

5x-y = 8

(b) Define simple Transformation and Quadratic transformation. Also show that f(x) is a Quasi concave (Quasi convex) on T_{++} and T_{--} (T_{+-} and T_{-+})

OR

(a) Define Hyper Geometric function.

(b) Prove that $2\,f_1\left(\frac{a,b;z}{c}\right) = (1-z)^{-a}\, 2\,f_1\left(\frac{a,c-b}{c}; \frac{z}{z-1}\right)$

Q.4 (a) Express $f(x) = x^4+2x^3+2x^2-x-2$ in Terms of Legendre's polynomial.

(b) Obtain rodrigne's formula for Legendra polynomial.

OR

(a) Prove that $\int_{-1}^{1} P_n(x)dx = 0$ if $n \geq 1$

(b) If P_n is a Legendre's polynomial of degree n then to prove the relation

$$(2n+1)xPn = (n+1)\,P_{n+1} + 1 + nP_{n-1} - 1$$

Q.5 (a) Prove that – $J'n(x) = J_{n-1}(x) - J_{n+1}(x)$

(b) Define Bessel's differential equation.

OR

(a) Show that

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi}}\left(\frac{\sin x}{x} - \cos x\right)$$

(b) Show that the Bessel functions $I_n(x)$ are linked together by relation-

$$e^{x/2\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$$